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Technical Report #3

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ABSTRACT: Some simple procedures are developed for studying the topologic and geometric properties of delta distributary systems. A delta channel network has three kinds of vertices (forks, junctions, and outlets) and six kinds of links, each corresponding to one of the six possible combinations of upstream and downstream vertices. Various functions of the vertex and link numbers may be used to specify the topologic properties of the network. A particularly useful function is the recombination factor, or ratio of number of junctions to number of forks. This ratio varies from zero for networks with no recombination to unity for braided streams.

A detailed toplogic study of the networks of five natural deltas (Colville, Irrawaddy, Yukon, Niger, and Parana) shows recombination factors ranging from 0.5 to 0.85. The frequency of different kinds of links can be explained reasonably well by a simple model that assumes random connection of vertices. The link lengths for a given network appear to belong to a common distribution and to depend relatively little on location with respect to the coast. The results on the Parana suggest that it should be considered as two deltas in tandem, each with its characteristic recombination factor.

INTRODUCTION

A channel network, as defined by Shreve (1966, p. 20), consists of the channels upstream from an arbitrarily chosen point (called the outlet) in a drainage network. Although the geometric structures of channel networks may differ widely from one to another, the topologic structures all have certain features in common. Each network is characterized by a given number of sources, from which single channels, or streams, are initiated. As the network develops downstream, the number of channels is reduced by successive combination of pairs of streams into one, until finally the outlet channel is created by the last two. Branching or dividing in the downstream direction does not occur (or is simply neglected if it occurs on a limited scale). The concept of a channel network has proved to be of both theoretical and practical importance in drainage basin analysis. Recent developments in the field have been comprehensively reviewed by Haggett and Chorley (1969).

Two other kinds of networks are important in drainage basin geomorphology: both involve the branching of single channels into two, as well as the combination process mentioned above. Braided channel networks are alternatives to the single-thread streams that constitute ordinary channel networks; they consist of multiple branching and reuniting (anastomosing) channels that begin with and terminate in a single stream. Delta channel networks are the distributary systems of deltas. They also start from a single channel, but the branching process predominates so that the typical fan-like form is developed, and the network terminates in multiple outlets at the coast. Clearly both braided and delta networks are topologically considerably more complex than ordinary channel networks. Howard et al. (1970) have recently made an extensive quantitative study of braided systems. In this paper, we present the results of a preliminary study of the quantitative properties f delta channel networks.

GENERAL PROCEDURES AND DEFINITIONS

Figure 1a shows an actual delta pattern and Figure 1b the channel network derived from it by procedures described below. First of all, only those channels below the delta source (bifurcation furthest upstream) are considered. Additional tributaries (such as _a_ of Figure 1a) are neglected, as are the small streams (_b_, _c_) that drain off into interior basins. These omissions appear to be justifiable as long as the physically important properties (e.g., discharge) are small compared to those of the mainstream. Channels that are depicted on the map by double lines are represented in Figure 1b by their center lines. In principle, it might be better to represent them by their thalwegs, but this information is rarely available. No more than three channels are allowed to join at a point. Apparent exceptions, such as _d_, must be resolved by more detailed mapping or by an arbitrary, but generally unimportant, decision.

A delta channel network of even moderate size contains so much detail that it would be awkward, if not self-defeating, to try to include all of it in a quantitative analysis. Consequently, in addition to the general procedures described above, each investigator may impose certain specific rules, germane to his own study, that are intended to eliminate unnecessary detail. Thus a hydrologist might choose to omit channel segments that carry less than some minimum discharge; a transportation expert might neglect all channels that have less than some minimum depth. Networks obtained from the same map by these two sets of rules could be quite different. As examples of special rules, in Figure 1b, those channels indicated on the map by only a single line (_c_, _e_) and the smallest island (_f_) are omitted.

The channel network of Figure 1b, an abstraction of the actual network of Figure 1a, is called in mathematics a _directed graph_. Although we do not make formal use of graph theory in our analysis, the general approach is guided by graph theoretical considerations. First, it is necessary to provide some reasonably precise definitions of the concepts employed. Points at which channels intersect or terminate are called _vertices_, and are indicated by dots in Figure 1b. A channel segment connecting two successive vertices is called a _link_. An area bounded by channels and having no channels in the interior is called an _enclosure_, and an area bounded by only two links (stream divides and recombines without any intervening vertices) is called a _simple enclosure_. The network of Figure 1b contains two enclosures, A and B, neither of which is

simple.

DERIVATION OF TOPOLOGIC PROPERTIES

Classification of Vertices

Probably the two most useful parameters in characterizing a delta network are the number of vertices, N_V, and the number of links, N_L. The network of Figure 1b has 13 vertices and 14 links. Note that there are three different kinds of vertices: forks (one channel divides into two), junctions (two channels combine to form one), and outlets (channel terminates). In Figure 1b, vertices 1, 3, 4, 5, 7, and 8 are forks, 2 and 6 are junctions, and 9 through 13 are outlets. The source vertex (1) is defined to be a fork, since it is fed by a single channel not included in the delta network. We have

$$N_V = N_F + N_J + N_O \tag{1}$$

This classfication of vertices is useful in developing a formal procedure for generating channel networks. First, the source vertex may be considered as generating two links, each of which terminates in a vertex. If one of these new vertices is a fork, two more links and vertices are produced. If it is a junction, it is located on the interior of an existing channel, thus increasing the number of links by one without changing the number of vertices. If it is an outlet, nothing new is added. For example, the network of Figure 1b can be generated by the following sequence: 1-5, 1-4, 4-3, 4-7, 3-2, 3-8, 7-6, 7-13, 5-9, 5-10, 8-11, 8-12, where the numbers identify the vertices at the upstream and downstream ends, respectively, of channel segments. We should perhaps make it clear that this sequence is not intended to have any relation to the chronological development of the actual network of Figure 1a. It is merely a formal recipe for reproducing the topologic properties of the network of Figure 1b.

Recapitulating, a fork generates two new links and two new vertices; a junction generates one link; and an outlet generates nothing. Thus we have

$$N_V = 2N_F + 1 \tag{2}$$

where the last term accounts for the source vertex. Then

$$N_F = \tfrac{1}{2}(N_V - 1) \tag{3}$$

$$N_J + N_O = \frac{1}{2}(N_V + 1) \tag{4}$$

Thus a delta channel network with N_V vertices has exactly $\frac{1}{2}(N_V - 1)$ forks, while the remaining $\frac{1}{2}(N_V + 1)$ vertices are divided between junctions and outlets. Note that N_V is always odd.

Our rules for generating a network also lead to

$$N_L = 2N_F + N_J \tag{5}$$

By combining (2) and (5), we find

$$N_J = 1 + N_L - N_V \tag{6}$$

The reader may easily verify that N_J is also the number of enclosures in the network. Incidentally, N_J is known in graph theory as the cyclomatic number. Howard et al. (1970) have used it in analyzing braided stream networks.

From (1), (3), and (6), we find

$$N_O = \frac{1}{2}(3N_V - 2N_L - 1) \tag{7}$$

The five variables N_V, N_L, N_F, N_J, and N_O, are related is such a way that if either of (N_V, N_F) and any one of (N_L, N_J, N_O) are known the other three can be found. In making determinations for large networks, it appears that counting N_V and N_O is generally fastest and least likely to produce errors.

A delta channel network is not required to have any junctions but it must have at least one outlet; thus the minimum possible values of N_J and N_O are zero and unity, respectively. A channel network with one source and one outlet really corresponds to a braided stream, but it may be considered as a limiting case of a delta network. The ranges of possible values of N_J, N_O, and N_L are as follows:

N_J: $0, 1, 2, \ldots, \frac{1}{2}(N_V - 1)$.

N_O: $\frac{1}{2}(N_V + 1), \frac{1}{2}(N_V - 1), \frac{1}{2}(N_V - 3), \ldots, 1$.

N_L: $N_V - 1, N_V, N_V + 1, \ldots, \frac{3}{2}(N_V - 1)$.

As stated before, selection of one variable determines the other two (as functions of N_V). For example, if $N_J = 1$, then $N_o = \frac{1}{2}(N_V - 1)$, and $N_L = N_V$.

It may be expected that various topologic properties of channel networks can be efficiently characterized by appropriate functions of these five parameters. In investigating several possibilities, we found one which appeared to be particularly useful. The recombination factor, α, is the ratio of number of junctions to number of forks (or maximum possible number of junctions).

$$\alpha = \frac{N_J}{N_F} = \frac{2N_J}{N_V - 1} \tag{8}$$

Alpha takes on values ranging from zero for a network with no junctions to unity for a braided stream. Examples of its application are given in the next section.

The results and ideas expressed in equation (1-8) could undoubtedly be inferred from one of the standard texts on graph theory, such as Berge (1962). The relevant theorems in such texts, however, seldom apply to our specific model, and thus require either generalization or specialization. It seemed preferable instead to develop the necessary relations by simple heuristic methods, as we have done.

Classification of Links

There are six kinds of links in a delta network - FF, FJ, JF, JJ, FO, and JO, where the letters identify the kinds of vertices at the upstream and downstream ends, respectively. The way in which the total number of links in a given network are distributed among these six classes depends in part on N_L and N_V but is not completely determined by them. This later point can easily be verified by constructing pairs of networks which have the same N_L and N_V but different link distributions; an example is shown in Figure 2(a and b). For quantitative relations, we have of course

$$n_{FF} + n_{FJ} + n_{JF} + n_{JJ} + n_{FO} + n_{JO} = N_L. \tag{9}$$

There are, however, more detailed conditions which the link numbers must satisfy.

$$n_{FF} + n_{FJ} + n_{FO} = 2N_F \quad \text{(10a)}$$

$$n_{JF} + n_{JJ} + n_{JO} = N_J \quad \text{(10b)}$$

$$n_{FF} + n_{JF} = N_F - 1 \quad \text{(10c)}$$

$$n_{FJ} + n_{JJ} = 2N_J \quad \text{(10d)}$$

$$n_{FO} + n_{JO} = N_O \quad \text{(10e)}$$

Equation 10a simply states that the total number of links originating at a fork is twice the number of forks; the other equations have similarly obvious interpretations. There are five equations in six unknowns, indicating, as stated above, that knowledge of N_L and N_V is not sufficient to completely determine the link numbers. (Additional relations that we might try to use, such as equation 9, are not independent of the set of five.) By employing the usual procedures, we can show that the coefficient and augmented matrices of equations 10a-e both have rank four, which means that four of the unknowns can be obtained as functions of the other two (but not any two). Specifically, complete solutions can be obtained if either of (n_{FO}, n_{JO}) and any one of the other four are known, or if n_{FJ} and either of (n_{FF}, n_{JF}) are known.

Less information is required for some special cases. For example, if there are no junctions ($\alpha = 0$), $n_{FF} = N_F - 1$, $n_{FO} = N_F + 1$, $n_{FJ} = n_{JF} = n_{JJ} = n_{JO} = 0$. This case is isomorphic to that of ordinary channel networks (Shreve, 1966). At the other extreme, if $\alpha = 1$, we find $n_{FO} = 0$, $n_{JO} = 1$, $n_{FF} = n_{JJ}$. Two of the unknowns can be determined if the third is given.

The fact that there are different link distributions for the same N_L and N_V suggests a statistical approach to the problem. Lacking a detailed statistical theory, we offer here a model which, though clearly oversimplified, is useful in exhibiting the broad features of link distributions. We assume that all vertices are connected at random, so that, for example, the fraction of FJ links is simply the product of the respective probabilities of finding a fork

and a junction. That is,

$$f_{FJ} = \frac{n_{FJ}}{N_L} = N_F N_J / N_V^2 \tag{11}$$

with analogous relations for the other kinds of links. Now delta networks are obviously not connected in a completely random fashion (for instance, all outlets tend to occur at roughly the same number of link traversals from the source), but it would appear that the model should be better for large networks than for small ones, simply because the number of ways of making connections increases very rapidly with N_V. Accordingly, we shall restrict the application of the model to networks above some minimum size, say $N_V > 50$.

A useful way to demonstrate the consequences of this model is to express N_J, N_O, and N_L as functions of α, calculate the expected link frequencies as functions of α and N_V and take the limits as N_V approaches infinity. The relative link frequencies can be expressed as fractions with a common denominator $(2 + \alpha)^2$ and numerators given by the matrix

	F	J	O
F	2	4α	$2(1 - \alpha)$
J	α	$2\alpha^2$	$\alpha(1 - \alpha)$

Thus $f_{FJ} = 4\alpha/(2 + \alpha)^2$.

These quantities are shown plotted against α in Figure 3. A few pertinent features may be noted. For values of α greater than $\frac{1}{3}$, which range appears to include most natural networks, the links are predominantly FF and FJ, with n_{FJ} greater than or less than n_{FF} depending on whether α is greater than or less than $\frac{1}{2}$. Two of the link numbers, n_{FF} and n_{FO}, decrease monotonically with increasing α while n_{FJ}, n_{JF} and n_{JJ} all increase monotonically. The fraction of JO links has a maximum (0.042) at $\alpha = 0.4$. This last observation is easily understood qualitatively; for small α, there are not many junctions and for large α, there are not many outlets.

As we have no way of making a quantitative assessment of the validity of

this model, its usefulness can be determined only by making a direct comparison with data on actual delta networks. This comparison is carried out in a later section, and as we shall see, the model, though often wrong in detail, provides a good general description of link distributions for natural networks.

The Connectivity Matrix

The preceding parts of this section have dealt with various kinds of topologic information on delta networks. All of this information, and, indeed, all topologic information about a given network, can be obtained from the connectivity matrix, C, a square matrix with N_V rows and columns. Rules for constructing C are as follows. If the i^{th} row and j^{th} vertices of the network are the upstream and downstream ends, respectively, of a link, then the element in the i^{th} row and j^{th} column of C is one. As a special case of this rule, if the i^{th} vertex is a fork whose branches recombine at j to form a simple enclosure, then the (i, j) element of C is two. All other elements of C are zero. The connectivity matrix for the network of Figure 1b is

$$C = \begin{array}{c|ccccccccccccc}
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 \\
\hline
1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
7 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\
9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
11 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
12 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
13 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array} \qquad (12)$$

The topologic properties of the network can be determined by simple manipulation of the matrix elements of C. N_V is of course the number of rows or columns; N_L is the sum of all matrix elements. Outlets, junctions, and forks can be identified by the fact that the sums along the corresponding rows are zero, one, and two, respectively. The link corresponding to the (i, j) position in C can be classified by identifying the kinds of vertices associated with the i^{th} and j^{th} rows.

The total number of different paths from the source to the coast (N_P) is a parameter of some interest in the analysis of natural delta networks (Coleman and Wright, private communication). For the extreme cases $\alpha = 0$ and $\alpha = 1$, the number of paths is just $N_F + 1$. For the intermediate cases, N_P is very sensitive to the exact topologic structure of the network and can be much greater than $N_F + 1$. Figure 2 shows two networks (b and c) which have the same vertex and link numbers but different N_P, so that we obviously cannot express N_P as a function of the previously derived topologic properties. We show how the connectivity matrix can be used to determine N_P for any given delta network. It is a well-known result of network theory that C^N gives the total number of N-link steps between pairs of vertices. If the longest path from the source to the outlet is M (in number of links traversed), then the sum $C + C^2 + C^3 + \ldots + C^M$ gives the total number of paths of any length between pairs of vertices. N_P is the sum of elements in the first row and in the columns corresponding to the outlets. For the matrix of equation 13, we find $M = 5$, $N_P = 9$, with 2, 2, 2, 2, and 1 paths to outlets 9, 10, 11, 12, and 13, respectively.

ANALYSIS OF NATURAL SYSTEMS

General Rules for Map Studies

In this section, we report the results of a topologic analysis of five natural deltas, the Colville, the Irrawaddy, the Niger, the Parana, and the Yukon. The selection of deltas to be studied was made almost entirely on the basis of availability of maps. The derived channel networks are shown in Figures 4a-e, and the maps from which they were obtained are listed in Table 1. The following general rules were used in obtaining the networks:

1. The study was limited to the active area of the delta. Table 1

lists the approximate locations of the delta sources. The most difficult decision on this point concerned the Irrawaddy, where the active area is rather tenuously connected to a much larger network.

2. We considered only those channels that were indicated on the map by double lines. An exception to this rule occurred for the Niger delta. The two maps used in that study were prepared with different conventions in representing the streams, so that double-line channels on NB-32 turn into single-line channels on NB-31. Thus, it was necessary to make a rather arbitrary selection of channels on NB-31; fortunately, most of the active area is on NB-32.
3. Islands with areas less than about 0.02 in^2 on the map were neglected. This corresponds to actual areas of 0.02, 0.31, and 5.0 mi^2 for scales of 1 to 63360, 250000, and 1000000, respectively.
4. Some transverse channels near the coast that appeared to have a tidal origin were ignored. This rule also resulted in some rather arbitrary decisions but we found that an appreciable range of alternate choices gave surprisingly small variation in topologic properties.
5. The completed networks have occasional links where it is difficult to determine the direction of flow. The choice in such cases does not affect the vertex distribution but it can affect the link distribution and can make a substantial difference in N_P.

Topologic Properties

Table 2 lists the topologic properties of the five delta networks. Most of the results were obtained via a computer program that takes the connectivity matrix as input. It should be noted, however, that they can also be obtained by direct counting methods with relatively little labor. A recommended procedure is to begin by counting N_V and N_O and then to calculate N_F, N_J, and N_L. In

determining the link numbers, if one set of interior links (FF, FJ, JF, or JJ) and one set of exterior links (FO or JO) is counted, the other four can be calculated from equations 10a-e. In practice, we have found it advisable to count two sets of interior links as a check against error. The counting can be greatly facilitated by color-coding the vertices. Perhaps the best way to obtain N_P is to begin at the source and count the number of paths to links that are successively further and further away. Whenever a fork is traversed, the number of paths does not change; whenever a junction is traversed, the number of paths to the link immediately downstream is just the sum of the respective number of paths to the two links immediately upstream.

Note that the values of α in Table 2 lie in the range 0.5 - 0.9. One common characteristic of four of the five cases is that if we consider successively larger subnetworks, each with the original source, the subnetworks quickly attain α-values comparable with that of the complete network and remain relatively constant thereafter. As an example, for the Yukon with subnetworks having N_V = 13, 25, 43, 63, 95, and 135, the α-values are 0.50, 0.67, 0.62, 0.71, 0.68, and 0.66, respectively; the Colville, the Irrawaddy, and the Niger behave similarly. For the Parana, however, α rises to 0.86 and then decreases steadily to 0.51 for the complete network. This variation in α correlates well with the geometry of the network shown in Figure 4e. For the first three-quarters of its length, the Parana delta remains confined to a relatively narrow region, rather more like a braided stream than a delta; in the last quarter, it begins to develop the typical delta shape.

The great variability in N_P is strikingly exhibited in Table 2. The Yukon and Niger deltas have very similar values of N_L and N_V, but N_P for the Yukon is about six times that for the Niger. A similar difference occurs in comparing the Irrawaddy with the Parana.

Table 3 gives the link distribution frequencies as obtained from the link number data of Table 2 and from Figure 3. We see that the random connection model gives surprisingly good results considering its simplicity and the lack of theoretical justification for it. The greatest relative error occurs in the prediction of f_{FO} and f_{JO} for the Colville, Irrawaddy, and Niger. In two cases, the observed values of f_{JO} are considerably greater than the theoretical maximum of 0.042.

It is not immediately obvious whether the deviations from the random connection model are in themselves random or whether they have some pattern. From physical considerations the most likely kind of deviation would be a non-randomness in the location of forks and junctions with respect to their distance from the source. In order to check this point, we define the topologic distance, λ, of a vertex from the source as the minimum number of links which must be traversed in the direction of flow in order to reach the vertex from the source. For example, the topologic distance for vertex 5 in Figure 1b is two. We have used the Kolmogorov-Smirnov test to determine whether there is any difference in the distributions of the topologic distances for forks and junctions, respectively, in the five deltas. The results are mixed. The Colville and Yukon deltas show no significant difference at the five percent level in the distributions of λ_F and λ_J. For the Irrawaddy and Niger deltas, the forks are located significantly nearer the source than are junctions; for the Parana, the reverse is true. This last observation can be correlated with the unusual structure of the Parana delta. Forks and junctions are distributed uniformly in the elongated region but there is a considerable predominance of forks in the area near the mouth.

Geometric Properties

Link lengths were measured with an architect's scale, the curved links being approximated by a series of straight-line segments. Measurements were made to the nearest 1/40 inch, corresponding to 0.025, 0.1 and 0.4 mi for scales of 1 to 63360, 250000, and 1000000, respectively. The absolute accuracy is of course somewhat less. Table 4 gives some descriptive statistics on link lengths for the five deltas. Comparison of means and standard deviations is of course significant only for networks taken from maps of the same scale. All five sets of links have roughly the same sort of right-skewed distribution, as indicated by columns four and five.

The length distributions of different kinds of links (FF, FJ, etc.) were compared by various non-parametric tests. The occurrence of significant differences at the five percent level was just about in the 1 out of 20 ratio that would be expected by chance if the null hypothesis holds. Thus, one may assume that all link lengths for a given network are taken from the same distribution.

The above result does not preclude the possibility that the length of a link may still depend on its location. In particular, one could look for correlation between link length and λ , but it seems preferable to use a geometric measure of distance in this case. We have tried several length parameters (for example, straight-line distance from source to upstream and of link, and channel distance from source to midpoint of link) and used the Spearman rank correlation test for comparing the behavior of length parameters and the corresponding link lengths. Again the frequency of rejection of the null hypothesis for four of the deltas was not greater than that expected from chance, but it is worth noting that the great majority of the correlation coefficients were negative. The Parana again proved to be an exception; as can be seen clearly from the map in Figure 4e, the link lengths in the region near the coast are appreciably shorter than those in remainder of the delta.

SUMMARY

In this paper, we have devised some simple basic procedures for the quantitative analysis of delta channel networks. These procedures were suggested by, but are generally different from, those developed by Horton, Strahler, Shreve, and others for ordinary channel networks. We should point out, however, that there are factors that suggest that this type of approach may be less productive, given the same effort, for delta networks than for ordinary networks. First, the topologic structure varies markedly with river stage, the maximum complexity occurring somewhere between low and flood stage. Howard et al. (1970) have noted this same problem in the analysis of braided networks. Also, for many deltas, the network properties for given stage are continually changing, with new channels being added and old ones abandoned. In this case, we are inclined to believe that that statistical distribution of the topologic and geometric properties probably remain relatively constant in time, but we have no evidence either for or against this speculation. Finally, in delta channels near the coast, the width is often of the same order of magnitude as the link length, a situation that makes the concepts of link and link length somewhat ambiguous. Without specific investigations of each point, it is not clear just how serious these difficulties are.

Our procedures for analyzing delta networks begin with simple counts of vertex and link numbers. There are three kinds of vertices (forks, junctions,

and outlets) and six kinds of links, each corresponding to one of the six possible combinations of upstream and downstream vertices. The numbers of different kinds of links and vertices help characterize the topologic properties of the networks; functions of these numbers, such as the recombination factor, may be used to specify particular features.

A quantitative study of five natural deltas show that their channel networks have certain topologic and geometric features in common. Each network appears to have a characteristic recombination factor that is attained after relatively few vertices have been developed. (The Parana, which is an apparent exception to this simple rule, can be considered as two deltas in tandem, with a different recombination factors.) The link distribution in deltas can be described reasonably well by a simple model that assumes random connection of vertices. The link lengths for a given network all appear to belong to a common distribution and to depend very little on geometric location.

We feel that these conclusions are well-substantiated for our sample of five deltas, and that they can be used as starting points for future research. (It would of course be dangerous to infer, without further study, that they represent general properties of all natural delta networks.) One specific topic of investigation suggested by our results is the correlation of the topologic and physical properties of deltas.

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Table 1. General Information

	Maps Used				Approximate Source Location	
Delta	Scale	Names	Numbers	Origin	Lat.	Long.
Colville	1:63,360	Harrison Bay (B-1), Alaska Harrison Bay (B-2), Alaska		USGS[a]	$70^{o}15'N$	$153^{o}43'W$
Irrawaddy	1:250,000	Bassein, Burma Ama, Burma Rangoon, Burma	NE 46-16 ND 46-4 NE 47-13	USAF[b]	$16^{o}38'N$	$95^{o}25'E$
Yukon	1:250,000	St. Michael, Alaska Kwiguk, Alaska		USGS	$62^{o}35'N$	$163^{o}57'W$
Niger	1:1,000,000	Lagos, West Africa Douala, West Africa	NB 31 NB 32	USAMS[c]	$5^{o}33'N$	$6^{o}33'E$
Parana	1:1,000,000	Rosario, South America Buenos Aires-Montevideo, South America	SI-20 SI-21	AGS[d]	$33^{o}10'S$	$60^{o}21'W$

a. U. S. Geological Survey

b. U. S. Air Force Aeronautical Chart and Information Center

c. U. S. Army Map Service

d. American Geographical Society

Table 2. Topologic Properties of Delta Networks

Property	Colville	Irrawaddy	Yukon	Niger	Parana
	Delta				
N_V	107	71	135	131	71
N_L	140	100	178	181	88
N_O	20	6	24	15	18
N_F	53	35	67	65	35
N_J	34	30	44	51	18
α	0.642	0.857	0.657	0.785	0.514
n_{FF}	37	25	47	49	27
n_{FJ}	61	43	69	78	28
n_{JF}	15	9	19	15	7
n_{JJ}	7	17	19	24	8
n_{FO}	8	2	18	3	15
n_{JO}	12	4	6	12	3
N_P	244	129	5038	815	623
N_P/N_V	2.28	1.82	37.3	6.22	8.78
N_P/N_O	12.2	21.5	210	54.3	34.6

Table 3. Link Distribution Frequencies

Kind of Link	Colville		Irrawaddy		Yukon		Niger		Parana	
	obs.	calc.	obs.	calc.	obs.	calc.	obs.	calc.	obs.	calc.
FF	0.26	0.29	0.25	0.25	0.26	0.28	0.27	0.26	0.31	0.32
FJ	0.43	0.37	0.43	0.42	0.39	0.37	0.43	0.40	0.32	0.33
JF	0.11	0.09	0.09	0.10	0.11	0.09	0.08	0.10	0.08	0.08
JJ	0.05	0.12	0.17	0.18	0.11	0.12	0.13	0.16	0.09	0.08
FO	0.06	0.10	0.02	0.03	0.10	0.10	0.02	0.06	0.17	0.15
JO	0.09	0.03	0.04	0.02	0.03	0.03	0.07	0.02	0.03	0.04

Table 4. Link Length Statistics

Delta	N_L	$\bar{\ell}$(mi)	σ(mi)	CV	% less than mean
Colville	140	1.01	0.85	0.84	66.5
Irrawaddy	100	6.0	5.1	0.85	58.0
Yukon	178	3.0	3.1	1.03	68.0
Niger	181	7.1	6.1	0.86	60.2
Parana	88	9.0	9.9	1.10	68.2

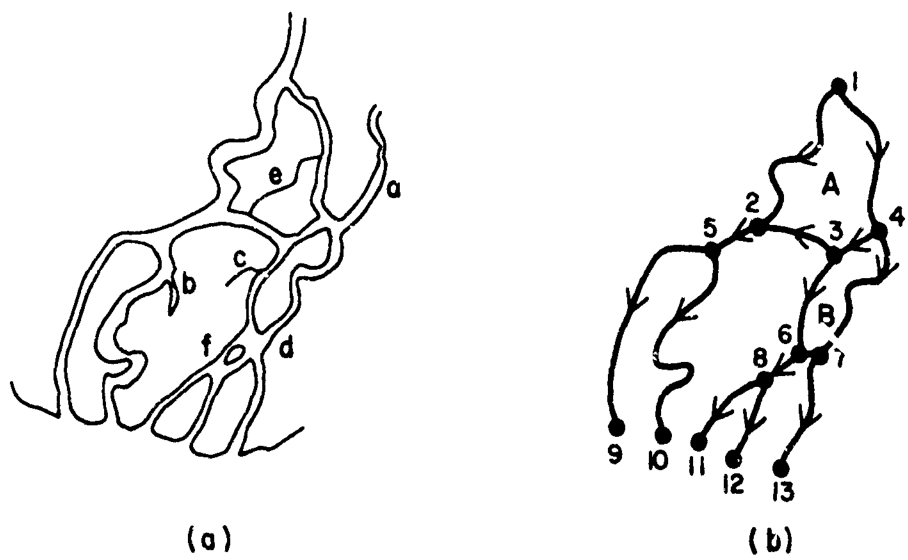
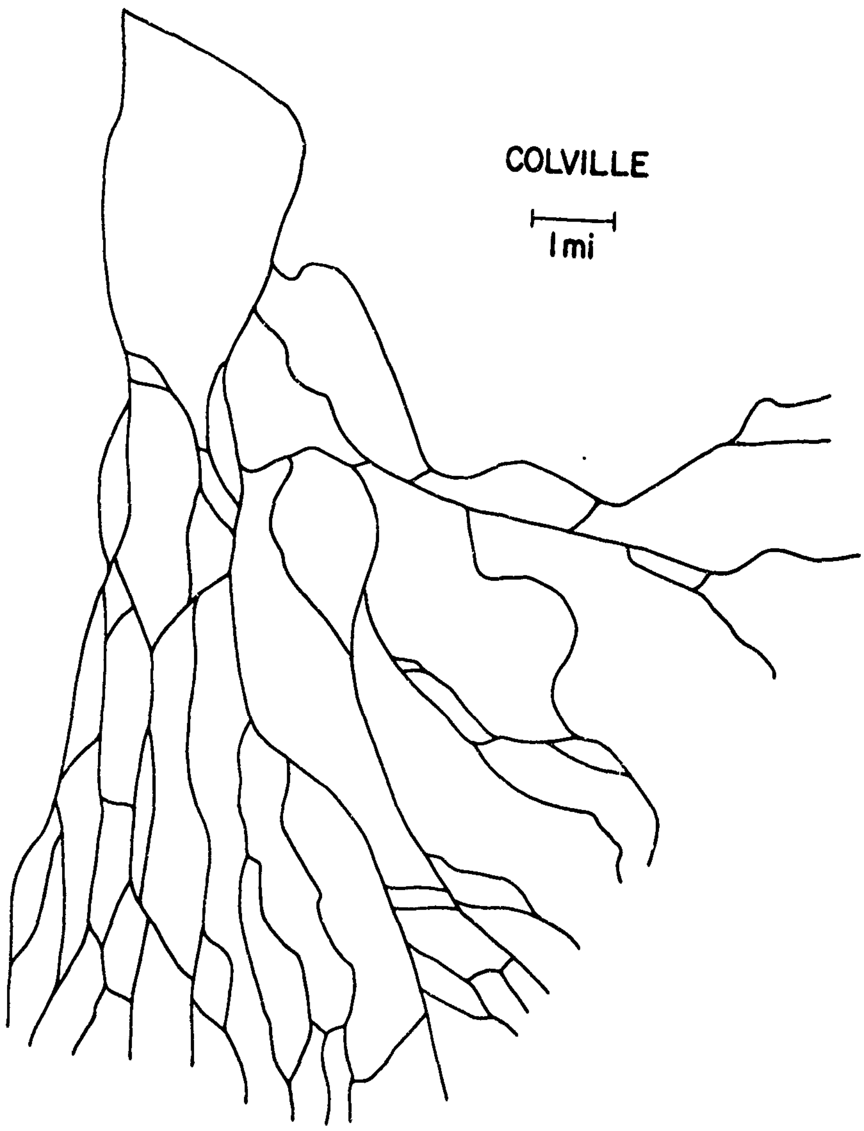
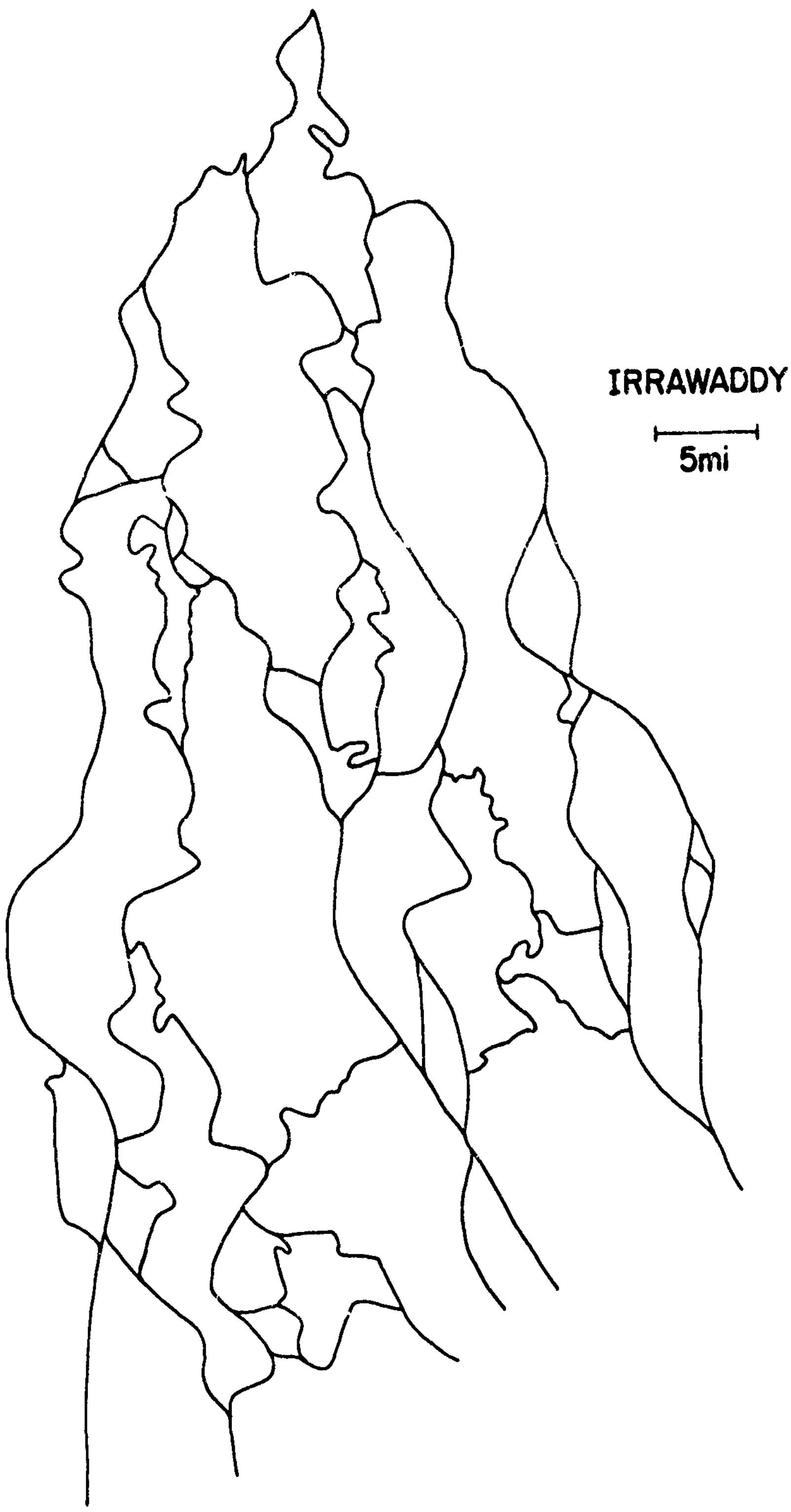


Figure 1(a). Outline of a natural delta network taken from a map. (b) The channel network derived from it. Letters and numbers are explained in the text.



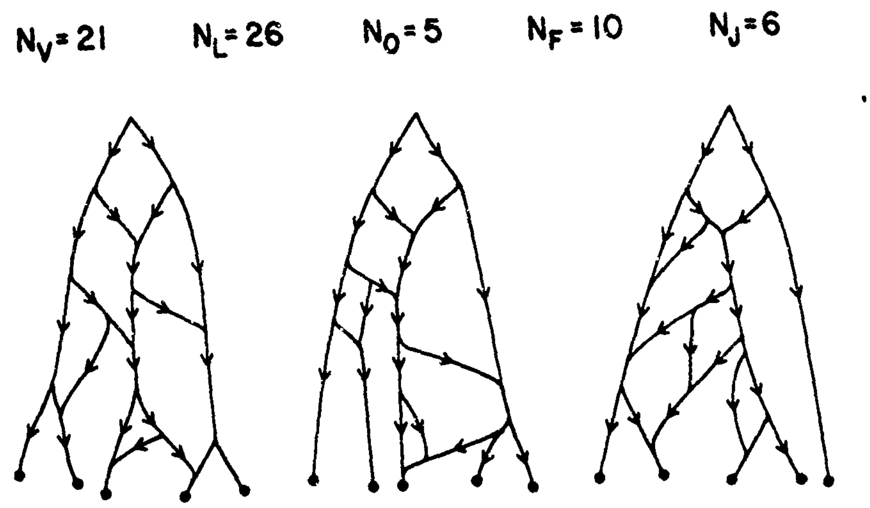
4(a)

Figure 4. Channel networks for five natural deltas: (a) Colville, (b) Irrawaddy, (c) Yukon, (d) Niger, (e) Parana.



4(b)

Figure 4. Channel networks for five natural deltas: (a) Colville, (b) Irrawaddy, (c) Yukon, (d) Niger, (e) Parana.



	(a)	(b)	(c)
n_{FF}	6	7	7
n_{FJ}	12	10	10
n_{JF}	3	2	2
n_{JJ}	0	2	2
n_{FO}	2	3	3
n_{JO}	3	2	2
N_P	18	21	19

Figure 2. Three hypothetical channel networks with the same numbers of links and vertices. (a) and (b) have different link distributions and different values of N_p. (b) and (c) have the same link distribution but different N_p.

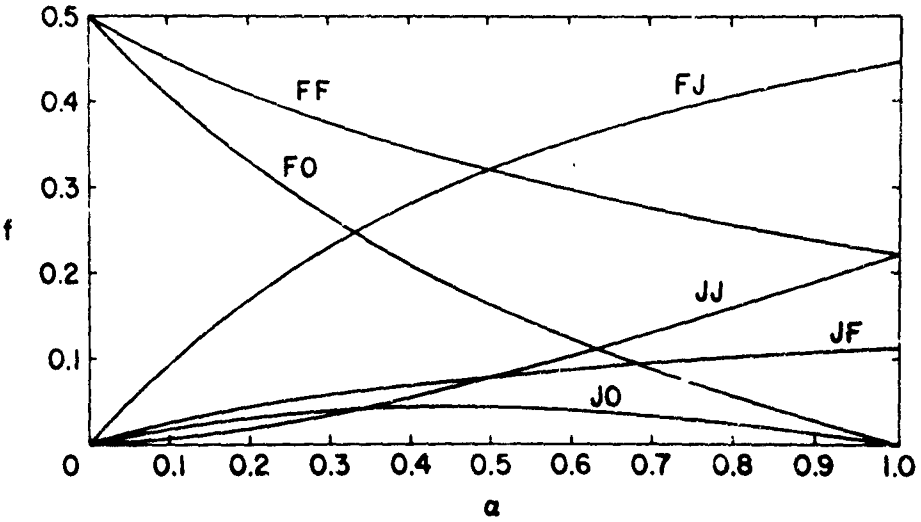
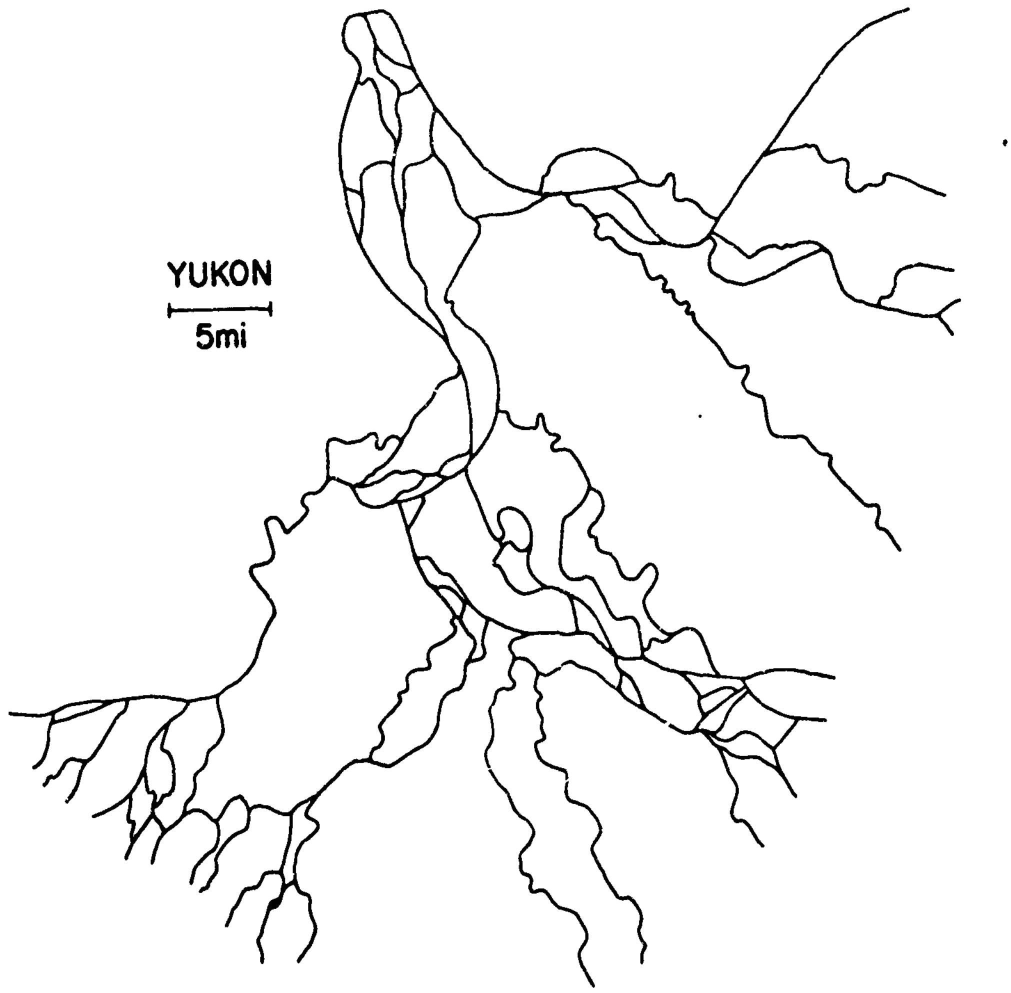
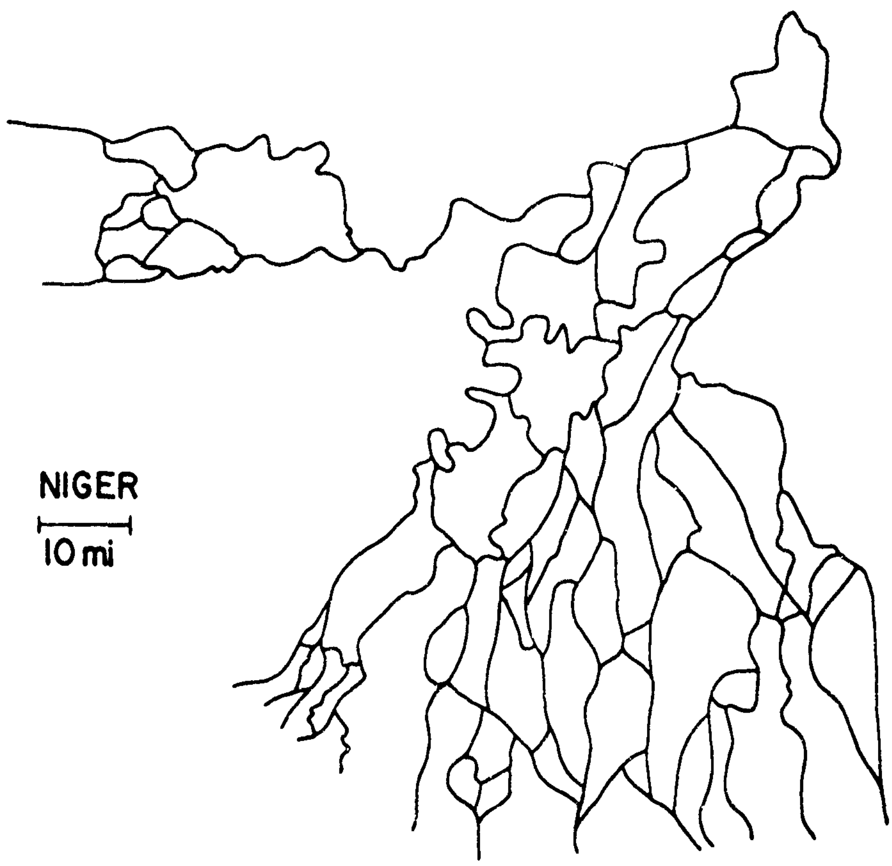


Figure 3. Relative link frequencies as a function of the recombination factor.



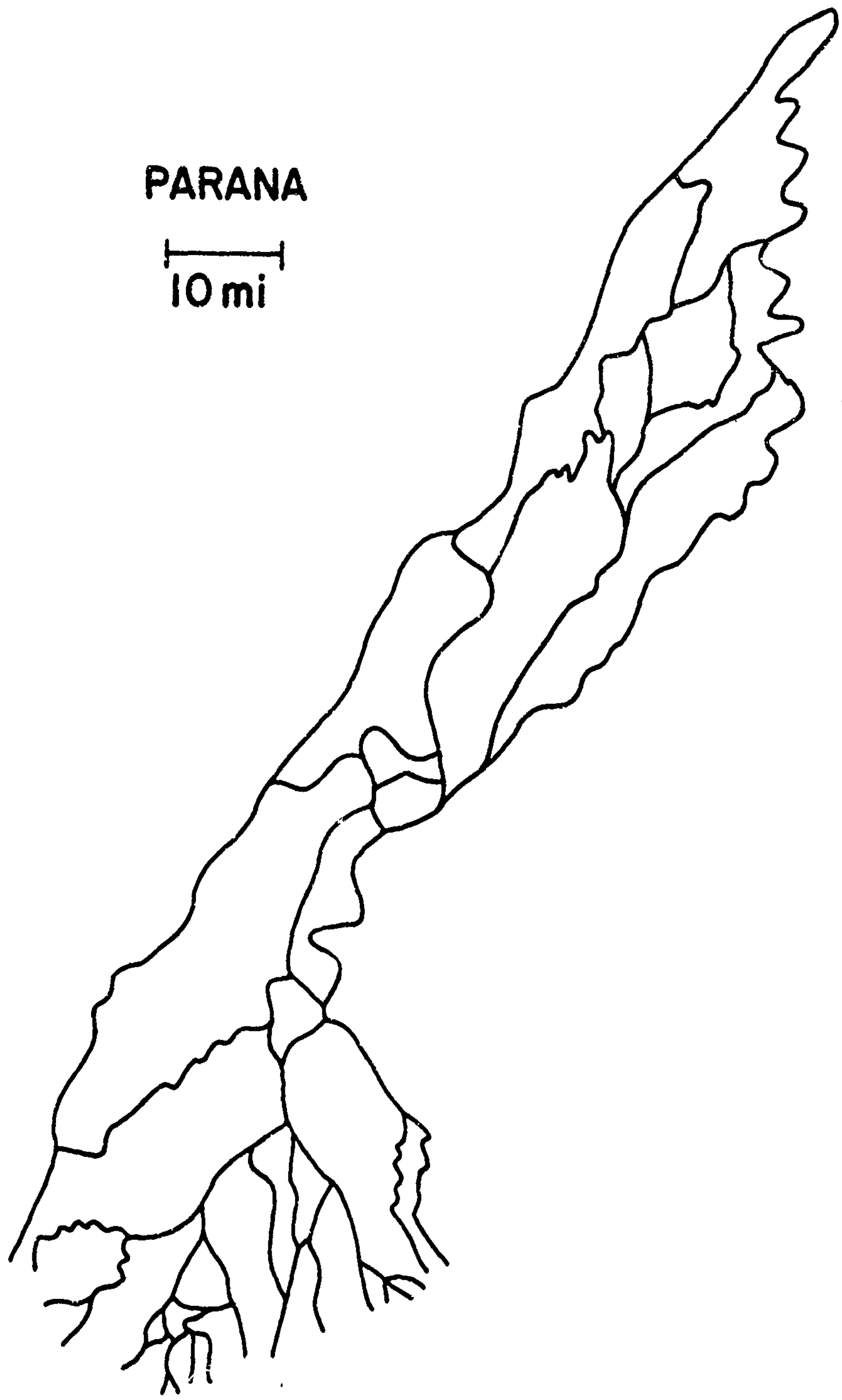
4(c)

Figure 4. Channel networks for five natural deltas: (a) Colville, (b) Irrawaddy, (c) Yukon, (d) Niger, (e) Parana.



4(d)

Figure 4. Channel networks for five natural deltas: (a) Colville, (b) Irrawaddy, (c) Yukon, (d) Niger, (e) Parana.



4(e)

Figure 4. Channel networks for five natural deltas: (a) Colville, (b) Irrawaddy, (c) Yukon, (d) Niger, (e) Parana.

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QUANTITATIVE PROPERTIES OF DELTA CHANNEL NETWORKS

Technical Report

J. Samuel Smart and Victor L. Moruzzi

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13 ABSTRACT

Some simple procedures are developed for studying the topologic and geometric properties of delta distributary systems. A delta channel network has three kinds of vertices (forks, junctions, and outlets) and six kinds of links, each corresponding to one of the six possible combinations of upstream and downstream vertices. Various functions of the vertex and link numbers may be used to specify the topologic properties of the network. A particularly useful function is the recombination factor, or ratio of number of junctions to number of forks. This ratio varies from zero for networks with no recombination to unity for braided streams.

A detailed topologic study of the networks of five natural deltas (Colville, Irrawaddy, Yukon, Niger, and Parana) shows recombination factors ranging from 0.5 to 0.85. The frequency of different kinds of links can be explained reasonably well be a simple model that assumes random connection of vertices. The link lengths for a given network appear to belong to a common distribution and to depend relatively little on location with respect to the coast. The results on the Parana suggest that it should be considered as two deltas in tandem, each with its characteristic recombination factor.

DD FORM 1473 (PAGE 1)

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Deltas						
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